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because $\sqrt{(y+a)^2}$ for him equals $y + a$. Indeed, since $y + a$ does not pass through zero, it might seem as though the root could be extracted in this way. In the numerator, however, there occurs the expression $y - a$ which does pass through zero when $y = a$ (*i. e.*, at the section through the focus) and this should be a danger signal.

There is another sign of trouble and a very fundamental one. We can not expect to get a single formula to express the attraction of the segment for the two cases $b < a$ and $b > a$, any more than we can expect to get a single formula for the attraction of a sphere on a point irrespective of whether the point is inside or outside of the sphere. The attraction is a continuous function of b; but the derivative of the force with respect to b is discontinuous when $b = a$, as is familiar to all who are acquainted with Poisson's Equation and its physical significance. The amount of the discontinuity is $4\pi\rho$.

The solution of this, and similar problems, is carried out in two steps. First when $b < a$, the attraction is toward the vertex of the paraboloid and equal to

$$A = 2\pi\rho \int_0^b \left(1 - \frac{a-y}{a+y}\right) dy = 4\pi\rho a \left[\frac{b}{a} - \log\left(\frac{b}{a} + 1\right)\right],$$

which, when $b = a$, gives the value $4\pi\rho a(1 - \log 2)$. When $b > a$, the calculation of the attraction of the part of the segment beyond the focus gives

$$2\pi\rho \int_a^b \left(1 - \frac{y-a}{y+a}\right) dy = 4\pi\rho a \left[\log\left(\frac{b}{a} + 1\right) - \log 2\right]$$

and directed away from the vertex. The whole segment from 0 to b, therefore, has the attraction equal to

$$A = 4\pi\rho a \left[1 - \log\left(\frac{b}{a} + 1\right)\right], \qquad b > a,$$

toward the vertex. This value vanishes when $b = a(e - 1)$ or when $b - a = (e - 2)a = 0.718a$. When b exceeds $a(e - 1)$ the resultant attraction is away from the vertex.

II. RELATING TO A PROBLEM IN MINIMA.

BY DUNHAM JACKSON, Harvard University.

On page 339 of Osgood's Calculus (Ex. 9), the following problem is proposed:

"Find the point so situated that the sum of its distances from the three vertices of an acute-angled triangle is a minimum."

The answer is given:

"The lines joining the point with the vertices make angles of 120° with one another."

Then the note is added,

"For a complete discussion of the problem for any triangle see Goursat-Hedrick, *Mathematical Analysis*, Vol. 1, § 62."

The following simpler proof, which the writer has not seen in print, is so very brief that it may be worth while to give space to it, in spite of the inherent probability that it has already appeared somewhere.[1]

In the first place, let P be a point free to move along a straight line, and O a fixed point not on the line; let r be the length of OP and θ the angle which it makes with the given line; and let x be the distance of P from a fixed point of the given line. Then it is seen immediately, either geometrically or analytically, that

$$\frac{dr}{dx} = \cos\theta,$$

except for the question of the algebraic sign, which is readily determined in any given case.

Now let ABC be the given triangle, and O the point, assumed to exist somewhere, which makes the sum of the three distances a minimum. Let the angles BOC, COA, AOB, be denoted by θ_1, θ_2, θ_3, respectively. Since the sum of the distances is less for O than for any other point of the plane, it is in particular less for O than for any other point of the line OA. Let O' be a variable point of OA, and x its distance from O, measured positively toward A. Let r_1, r_2, r_3 stand for the distances $O'A$, $O'B$, $O'C$ respectively, and θ_1', θ_2', θ_3' for the angles $BO'C$, $CO'A$, $AO'B$. By the remark above,

$$\frac{dr_1}{dx} = -1, \qquad \frac{dr_2}{dx} = -\cos\theta_3', \qquad \frac{dr_3}{dx} = -\cos\theta_2';$$

as to the signs, it is clear that r_1 diminishes when x increases,[2] while r_2 and r_3 diminish or increase according as the respective angles are acute or obtuse. When O' coincides with O, so that θ_2' and θ_3' reduce to θ_2 and θ_3, the sum of the three distances is to be a minimum, and the sum of the derivatives must be zero. That is,

$$\cos\theta_2 + \cos\theta_3 + 1 = 0. \tag{1}$$

If we apply the same reasoning to the lines OB, OC, we find

$$\cos\theta_3 + \cos\theta_1 + 1 = 0, \tag{2}$$

$$\cos\theta_1 + \cos\theta_2 + 1 = 0. \tag{3}$$

By addition of these three equations and division by 2,

$$\cos\theta_1 + \cos\theta_2 + \cos\theta_3 + \frac{3}{2} = 0. \tag{4}$$

Subtracting (1), (2), and (3) in turn from (4), we obtain

$$\cos\theta_1 = -\frac{1}{2}, \qquad \cos\theta_2 = -\frac{1}{2}, \qquad \cos\theta_3 = -\frac{1}{2}.$$

This is equivalent to the statement to be proved.

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[1] It is equivalent in substance to the proof given in de la Vallée Poussin's *Cours d'analyse infinitésimale*, Vol. 1, end of Chapter III; but it is slightly more elementary in form.

[2] If O' passes beyond the vertex A, of course r_1 increases with x; but it is not necessary to consider such points O'.

It will be noticed that in this demonstration no use has been made of the assumption that the angles of the triangle are acute. On the other hand, if an angle of the triangle is greater than 120°, it is clear that there is no point O such that the lines OA, OB, OC make angles of 120° with each other. The explanation of the paradox is, that it was tacitly assumed that O does not coincide with a vertex, and in the case of an angle greater than 120° the point which gives the minimum actually is the vertex of the obtuse angle.

NOTES AND NEWS.

SEND COMMUNICATIONS TO D. A. ROTHROCK, Indiana University, Bloomington, Ind.

The Royal Society recently granted the Sylvester Medal to Professor GASTON DARBOUX for his contributions to mathematical science.

Professor W. R. SMITH, of the University of Minnesota, has gone to Dallas, Texas, as teacher of mathematics in the Terrill School for Boys.

Dr. GILLIE A. LAREW, who has been on leave during the past year, has returned to her position in Randolph-Macon Woman's College as adjunct professor of mathematics.

Dr. ISAAC SHARPLESS, formerly instructor and professor of mathematics at Haverford College, and president since 1887, retires from active service at the end of the present college year.

The following appointments to instructorships in mathematics have recently been made: Mr. H. C. CLEVENGER, at the school of mines of the University of Montana; Mr. C. V. REYNOLDS, at Wesleyan University; and Mr. F. L. SMITH, at Princeton University.

Promotions have been announced as follows: Dr. W. M. SMITH, to a professorship of mathematics at Lafayette College; Assistant Professor R. M. WINGER, to a professorship at the University of Oregon; Dr. J. K. LAMOND, to a professorship at Pennsylvania College; Mr. H. W. MYERS, to a professorship at Huron College; Dr. H. W. ROEVER, to an associate professorship of mathematics and Dr. G. O. JAMES, to an associate professorship of mechanics and astronomy, at Washington University; Dr. R. L. MOORE, to an assistant professorship at the University of Pennsylvania; Dr. W. E. MILNE, to an assistant professorship at Bowdoin College; Dr. A. R. CRATHORNE, to an assistant professorship at the University of Illinois; Dr. J. J. LUCK, to an adjunct professorship of mathematics at the University of Virginia; and Dr. W. V. LOVITT and Dr. T. E. MASON, to assistant professorships at Purdue University.

Professor H. B. LEONARD, of the University of Oregon, becomes professor of mathematics at the University of Arizona, and assistant professor G. H. CRESSE has been promoted to an associate professorship of mathematics.